Test (2)

Mr. D

1. $\dfrac{80 - 6\left(\dfrac{36}{9}\right)}{0.25} =$

(a) 416 (c) 188
(b) 224 (d) 104

① $\dfrac{80 - 6 \times \left(\dfrac{36}{9}\right)}{0.25}$ =

$= \dfrac{80 - 6 \times (4)}{0.25} = \dfrac{80 - 24}{1/4}$

$= \dfrac{56}{1/4} = 56 \times \dfrac{4}{1} = 56 \times 4$

$= \boxed{224} \rightarrow \boxed{b}$

②
$$\begin{array}{r} 56 \\ \times\ 4 \\ \hline 224 \end{array}$$

2. If $27^{m} \times 3^{2} = 3^{4} \times 9^{8}$, then $m =$

(a) 3
(b) 6
(c) 8
(d) 15

2 $(27)^{m} \times 3^{2} = 3^{4} \times 9^{8}$

$(3^{3})^{m} \times 3^{2} = 3^{4} \times (3^{2})^{8}$

$3^{3m} \times 3^{2} = 3^{4} \times 3^{16}$

$\boxed{3}^{3m+2} = \boxed{3}^{20}$

$3m + 2 = 20$

$3m = 18 \rightarrow m = \boxed{6} \rightarrow$ b

3. If the product of two numbers is 5 and one of the numbers is $\frac{3}{2}$, then the sum of the numbers is:

(a) $4\frac{1}{3}$

(b) $4\frac{2}{3}$

(c) $4\frac{5}{6}$

(d) $5\frac{1}{6}$

3 $\frac{3}{2} \times \boxed{?} = 5$

$\boxed{?} = 5 \div \frac{3}{2} = 5 \times \frac{2}{3}$

$\boxed{?} = \frac{10}{3}$

the two numbers are: $\frac{3}{2}$ and $\frac{10}{3}$

sum $= \frac{3^{\times 3}}{2_{\times 3}} + \frac{10^{\times 2}}{3_{\times 2}}$

$= \frac{9}{6} + \frac{20}{6} = \frac{29}{6} = \boxed{4\frac{5}{6}}$

$\boxed{C}$

4. $\dfrac{x^2}{\sqrt{1-x^2}} - \sqrt{1-x^2} =$

(a) $\dfrac{-1}{\sqrt{1-x^2}}$

(b) $\dfrac{1}{\sqrt{1-x^2}}$

(c) $\dfrac{1-2x^2}{\sqrt{1-x^2}}$

(d) $\dfrac{2x^2-1}{\sqrt{1-x^2}}$

(4) $\dfrac{x^2}{\sqrt{1-x^2}} - \dfrac{\sqrt{1-x^2}}{1} \dfrac{\times\sqrt{1-x^2}}{\times\sqrt{1-x^2}}$

$= \dfrac{x^2}{\sqrt{1-x^2}} - \dfrac{1-x^2}{\sqrt{1-x^2}}$

$= \dfrac{x^2-(1-x^2)}{\sqrt{1-x^2}} = \dfrac{x^2-1+x^2}{\sqrt{1-x^2}}$

$= \boxed{\dfrac{2x^2-1}{\sqrt{1-x^2}}} \longrightarrow$ d

5. Consider the equation $x^2 + 2x + k = 5$, where k is a constant. If 3 is a solution of the equation, then the second solution is:

(a) -5
(b) -2
(c) -1
(d) -3

(5) Since $x = 3$ is a solution, we can substitute by $x = 3$

$$(3)^2 + 2(3) + K = 5$$
$$9 + 6 + K = 5$$
$$15 + K = 5 \longrightarrow K = -10$$

$$x^2 + 2x - 10 = 5$$
$$x^2 + 2x - 15 = 0$$
$$(x+5)(x-3) = 0$$
$$x = -5 \quad \text{or} \quad x = 3$$

the other solution is $\boxed{x = -5}$

a

6. The length of the diagonal of a square is $\sqrt{10}$. Find the area of the square.

(a) 40

(b) 20

(c) 10

(d) 5

(6) Area of square = $\rightarrow$ $(\text{side})^2$

$\rightarrow$ $\frac{1}{2}(\text{diagonal})^2$

$$A = \frac{1}{2}d^2 = \frac{1}{2} \times (\sqrt{10})^2$$

$$= \frac{1}{2} \times 10 = \boxed{5} \longrightarrow \text{d}$$

7. Cube A has surface area 1350 cm^2, and cube B has surface area 600 cm^2. Then the edge of A exceeds the edge of B by:

(a) 25
(b) 15
(c) 5
(d) None of the previous

(7)

S.A. of cube $= 6 \times (\text{side})^2$
V. of cube $= (\text{side})^3$

$$S.A = 6x^2$$
$$1350 = 6x^2$$
$$\frac{1350}{6} = x^2$$
$$x^2 = 225$$
$$x = \sqrt{225}$$
$$x = 15 \text{ cm}$$

$$S.A = 6y^2$$
$$600 = 6y^2$$
$$\frac{600}{6} = y^2$$
$$100 = y^2$$
$$y = 10 \text{ cm}$$

so x is 5 more than y $\longrightarrow$ $\boxed{C}$

8. $\dfrac{4x^3 - 2x}{2x+1} =$

(a) $2x^2 + x + \dfrac{1}{2} - \dfrac{\frac{1}{2}}{2x+1}$

(b) $2x^2 - x - \dfrac{1}{2} + \dfrac{\frac{1}{2}}{2x+1}$

(c) $2x^2 + x - \dfrac{1}{2} + \dfrac{\frac{1}{2}}{2x+1}$

(d) $2x^2 - x + \dfrac{1}{2} - \dfrac{\frac{1}{2}}{2x+1}$

8

$$
\begin{array}{r|l}
 & 2x^2 - x - \frac{1}{2} \\
\hline
2x+1 & \cancel{4x^3} + 0 - 2x \\
 & \ominus \cancel{4x^3} \mp 2x^2 \\
\hline
 & \cancel{-2x^2} - 2x \\
 & \oplus \cancel{-2x^2} \oplus\!- x \\
\hline
 & \cancel{-x} \\
 & \oplus \cancel{-x} \oplus\!- \frac{1}{2} \\
\hline
 & \frac{1}{2}
\end{array}
$$

Ans. $= \boxed{\left(2x^2 - x - \frac{1}{2}\right) \text{ Remainder } \frac{1}{2}}$

$= \boxed{2x^2 - x - \frac{1}{2} + \dfrac{1/2}{2x+1}} \longrightarrow$ b

9. Which of the following inequalities is equivalent to $-4 < x < 8$:

(a) $|x-1| < 7$

(b) $|x+2| < 6$

(c) $|x+3| < 5$

(d) $|x-2| < 6$

9 If we solve $|x-2| < 6$

$$-6 < x-2 < 6$$

$$-6+2 < x < 6+2$$

$$-4 < x < 8$$ ✓

$\boxed{d}$

10. The solution set of $\frac{1}{x^2}+\frac{1}{x}-12=0$ is:

(a) $\{2\sqrt{2}, \sqrt{3}\}$

(b) $\{2\sqrt{2}\}$

(c) $\left\{-\frac{1}{4}, \frac{1}{3}\right\}$

(d) None of the previous

10 $\frac{1}{x^2}+\frac{1}{x}-12=0$

$\times(x^2)$

$1+x-12x^2=0$

$-12x^2+x+1=0$

$\times(-1)$

$12x^2-x-1=0$

$(4x+1)(3x-1)=0$

or

$4x+1=0$ | $3x-1=0$

$x=\frac{-1}{4}$ | $x=\frac{1}{3} \rightarrow \boxed{\left\{\frac{-1}{4}, \frac{1}{3}\right\}}$

C

11. If $y = \dfrac{x}{1 - xz}$, then $z =$

(a) $\dfrac{1}{x}$

(b) $\dfrac{x}{1 - xy}$

(c) $\dfrac{1}{xy}$

(d) $\dfrac{y - x}{xy}$

11

$$y = \frac{x}{1 - xz}$$

$\times$

$$y(1 - xz) = x$$

$$y - xyz = x$$

$$y - x = xyz$$

$\div$

$$\frac{y - x}{xy} = z$$

$$\boxed{z = \frac{y - x}{xy}}$$

d

12. The solution set of $\left|\frac{x}{3}\right| > \frac{1}{2}$ is:

(a) $(-\infty, -6) \cup (6, \infty)$

(b) $(-6, 6)$

(c) $\left(\frac{3}{2}, \infty\right)$

(d) None of the previous

(12) $\left|\frac{x}{3}\right| > \frac{1}{2}$

$\frac{x}{3} > \frac{1}{2}$ <u>or</u> $\frac{x}{3} < \frac{-1}{2}$

$x > \frac{3}{2}$ <u>or</u> $x < \frac{-3}{2}$

Ans. is $\boxed{(-\infty, -\frac{3}{2}) \cup (\frac{3}{2}, \infty)}$

(d)

other form: $\boxed{\mathbb{R} / [-\frac{3}{2}, \frac{3}{2}]}$

13. If $f(x) = \begin{cases} x-1 & \text{if } x \geq 3 \\ 3-x^2 & \text{if } x < 3 \end{cases}$, then find $f(8) + f(-1)$.

(a) 9

(b) 11

(c) 5

(d) –5

13

$f(8) = 8 - 1 = 7$

$f(-1) = 3 - (-1)^2 = 3 - 1 = 2$

$f(8) + f(-1) = 7 + 2 = 9 \rightarrow$ a

14. If $f(x)=\begin{cases}\dfrac{1}{x-3} & , x<-1\\ \dfrac{\sqrt{1-x}}{x} & , x>1\end{cases}$, then find the domain of f.

(a) $\Re \setminus \{0,3\}$

(b) ϕ

(c) $(-\infty,-1)\cup(1,\infty)$

(d) None of the previous

(14) when $x<-1$

$x-3 \neq 0$

$x \neq 3$

$(-\infty,-1)$

when $x>1$

$x \neq 0$ and $1-x \geq 0$

$-x \geq -1$

$x \leq 1$

not possible

the domain is $\boxed{(-\infty,-1)} \rightarrow$ d

15. The price of copper increased by 25% and then fell by 20%. The price after these changes becomes.

(a) 5% less than the original price.
(b) 5% more than the original price.
(c) Same as original price
(d) None of the previous

15 let the original price is 100

$$100 + 25\% \text{ of } 100 = 100 + \frac{25}{\cancel{100}} \times \cancel{100}$$

$$= 100 + 25$$

$$= 125$$

$$125 - 20\% \text{ of } 125$$

$$= 125 - \frac{20}{\cancel{100}} \times \cancel{125} \quad \div 25$$

$$\div 25$$

$$= 125 - \frac{\cancel{20}}{\cancel{4}} \times 5$$

$$= 125 - 5 \times 5 = 125 - 25 = 100$$

the same $\rightarrow$ C

16. If 6 percent of x is 7.5, then 36 percent of x equals:

(a) 36
(b) 42
(c) 45
(d) 48

% : #

$6 : 7.5$

$36 : \boxed{?}$

$\boxed{?} = \frac{7.5 \times 36}{6}$

$= 7.5 \times 6$

$= 45.0$

$= \boxed{45} \rightarrow C$

3

$$\begin{array}{r} 7.5 \\ \times\ 6 \\ \hline 45.0 \end{array}$$

17. The weight of Sami was 100 kg. He started a diet that guarantees a 10% weight loss per month. What was Sami's weight after following this diet for two months?

(a) 80 kg
(b) 79 kg
(c) 81 kg
(d) None of the previous

17

1st. month: $100 - 10\%\text{ of }100 = 100 - \frac{\cancel{10}}{\cancel{100}} \times \cancel{100}$ (10 over 100 times 100)

$= 100 - 10 = 90$

2nd. month: $90 - 10\%\text{ of }90$

$= 90 - \frac{\cancel{10}}{\cancel{100}} \times 9\cancel{0} = 90 - 9$

$= 81$

C

18. In an Arabic school, English and French are offered as foreign languages, and each student must study at least one foreign language. If 41 students study both English and French, 681 students study English and 357 students study French, find the number of students in the school.

(a) 1079
(b) 1038
(c) 997
(d) 993

18

$$\boxed{n(F \text{ or } E) = n(F) + n(E) - n(F \cap E)}$$

$$n(F \text{ or } E) = 357 + 681 - 41$$
$$= 1038 - 41$$
$$= 997 \rightarrow C$$

$$\begin{array}{r} \scriptstyle 1 \\ 357 \\ +\ 681 \\ \hline 1038 \end{array}$$

$$\begin{array}{r} \scriptstyle 9\ 13 \\ \cancel{1}0\cancel{3}8 \\ -\ \ \ 41 \\ \hline 997 \end{array}$$

19. A water tank is half full of water. When 10 gallons are added, the tank is $\frac{7}{8}$ full. What is the capacity of the tank in gallons?

(a) $26\frac{2}{3}$

(b) $24\frac{3}{8}$

(c) $28\frac{1}{8}$

(d) $24\frac{2}{3}$

(19) let the full tank be $\boxed{x}$

$$\frac{1}{2}x + 10 = \frac{7}{8}x$$

$$10 = \frac{7}{8}x - \frac{1\times4}{2\times4}x$$

$$10 = \frac{7}{8}x - \frac{4}{8}x$$

$$10 = \frac{3}{8}x$$

÷

$$x = 10 \div \frac{3}{8} \rightarrow x = 10 \times \frac{8}{3}$$

$$x = \frac{80}{3} = \boxed{26\frac{2}{3}} \rightarrow a$$

20. The solution set of $|x + 1| = x + 1$ is:

(a) $\{0\}$
(b) $\{1\}$
(c) $\Re$
(d) $[-1, \infty)$

20

$$|x+1| = \begin{cases} x+1 & , x+1 \geq 0 \\ -(x+1) & , x+1 < 0 \end{cases}$$

$$|x+1| = \begin{cases} x+1 & , x \geq -1 \\ -(x+1) & , x < -1 \end{cases}$$

so $|x+1| = x+1$ when $x \geq -1$

$\boxed{[-1, \infty)} \rightarrow$ d