## Cambridge IGCSE ${ }^{\text {TM }}$

## MATHEMATICS

0580/41
Paper 4 (Extended)
October/November 2023
MARK SCHEME
Maximum Mark: 130

## Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.
Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

## GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.


## GENERIC MARKING PRINCIPLE 2:

Marks awarded are always whole marks (not half marks, or other fractions).

## GENERIC MARKING PRINCIPLE 3:

Marks must be awarded positively:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.


## GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

## GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

## Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

## Abbreviations

| cao | correct answer only |
| :--- | :--- |
| dep | dependent |
| FT | follow through after error |
| isw | ignore subsequent working |
| oe | or equivalent |
| SC | Special Case |
| nfww | not from wrong working |
| soi | seen or implied |


| Question | Answer | Marks | Partial Marks |
| :---: | :---: | :---: | :---: |
| 1(a)(i) | Translation $\binom{-7}{-1}$ oe | 2 | B1 for each |
| 1(a)(ii) | Rotation <br> $90^{\circ}$ clockwise oe $(5,1)$ | 3 | B1 for each |
| 1(b)(i) | $\begin{aligned} & \text { Image at } \\ & \quad(2,6)(3,6)(3,8) \end{aligned}$ | 2 | B1 for reflection in $y=k, k \neq 2$ or for reflection in $x=2$ |
| 1(b)(ii) | Image at $(-4,4)(-6,4)(-6,8)$ | 2 | B1 for an enlargement, sf -2 in the wrong position |
| 2(a) | 1960 | 2 | M1 for $\frac{1}{2} \times 9.8 \times 20^{2}$ oe |
| 2(b) | 1.5 or $11 / 2$ or $\frac{3}{2}$ | 3 | M1 for a first correct step, e.g. $20 y-15=15 \quad \text { or } \quad 4 y-3=3$ <br> M1FTdep for a second correct step, e.g. $20 y=30$ <br> or $4 y=6$ <br> or $y-\frac{15}{20}=\frac{15}{20}$ oe |
| 2(c) | $9 x-10$ final answer | 2 | B1 for $k x-10$ or $9 x+c$ <br> or M1 for $15 x-24$ or $-6 x+14$ <br> or $\mathbf{B 1}$ for correct answer seen and then spoiled |
| 2(d) | $\sqrt[3]{\frac{2 b^{2}-A}{3}}$ oe final answer | 3 | M1 for isolating $3 c^{3}, 3 c^{3}=2 b^{2}-A$ oe or for $\frac{A}{3}=\frac{2 b^{2}}{3}-c^{3} \text { or } \frac{A}{-3}=\frac{2 b^{2}}{-3}+c^{3}$ <br> M1FT for isolating $c^{3}$, follow through their first step dep on a 3-term expression with a $\mathrm{kc}^{3}$ term <br> M1FT taking the cube root to the final answer, follow through their previous step Maximum of two marks if answer incorrect |
| 2(e) | $\begin{aligned} & (2 q-1)(3 p-2) \text { or }(1-2 q)(2- \\ & 3 p) \text { final answer } \end{aligned}$ | 2 | M1 for $2 q(3 p-2)-[1](3 p-2)$ <br> or $3 p(2 q-1)-2(2 q-1)$ <br> or for correct answer seen then spoiled |
| 3(a)(i) | 227900000 | 1 |  |


| Question | Answer | Marks | Partial Marks |
| :---: | :---: | :---: | :---: |
| 3(a)(ii) | 51200 or 51190 or 51194 | 2 | $\text { M1 for } \frac{35.8}{100} \times 143000$ <br> After 0 scored SC1 for answer figs 512 or figs 5119 or figs 51194 |
| 3(a)(iii) | 2.43 or 2.434... | 1 |  |
| 3(a)(iv) | 3000 or 3004 to 3005 | 2 | M1 for $\frac{4.495 \times 10^{9}}{1.496 \times 10^{8}}[\times 100]$ oe <br> After 0 scored SC1 for answer figs 3 or figs 3004.... or figs 3005 |
| 3(a)(v) | 1.52 or 1.522... | 2 | B1 for $1 \mathrm{AU}=1.5[0] \times 10^{8}$ or $1.497 \ldots \times 10^{8}[\mathrm{~km}]$ or $1 \mathrm{~km}=6.68 \times 10^{-9} \text { or } 6.678 \ldots \times 10^{-9}[\mathrm{AU}]$ <br> OR <br> M1 for $\frac{5.2 \times 2.279\left[\times 10^{8}\right]}{7.786\left[\times 10^{8}\right]}$ oe <br> After 0 scored SC1 for answer figs 152 or figs 1522...... |
| 3(a)(vi) | 4890 or 4885... | 2 | M1 for $d \times\left(1+\frac{39.2}{100}\right)=6800$ oe |
| 3(b)(i) | $\begin{aligned} & 2.9979 \times 10^{5} \times 60^{2} \times 24 \times \\ & 365.25 \end{aligned}$ | M1 | After M0 <br> SC1 for $2.9979 \times 10^{5} \times 31557600$ oe |
|  | $=9.4606 \ldots \times 10^{12}$ | A1 |  |
| 3(b)(ii) | 2.54 or 2.536 to 2.537 | 2 | M1 for $\frac{2.4 \times 10^{19}}{9.461 \times 10^{12}}$ oe |
| 4(a)(i) | $\frac{2}{5} \quad \frac{5}{9}, \frac{4}{9} \quad \frac{5}{9}, \frac{4}{9}$ | 2 | B1 for $\frac{2}{5}$ and a pair of probabilities for spinner B that sum to 1 |
| 4a(ii)(a) | $\frac{1}{3} \text { oe }$ | 2 | FT dep their tree diagram M1 for $\frac{3}{5} \times$ their $\frac{5}{9}$ |
| 4a(ii)(b) | $\frac{2}{3} \text { oe }$ | 1 | $\text { FT dep } 1-\text { their } \frac{1}{3}$ |
| 4(b) | 72 | 1 |  |


| Question | Answer | Marks | Partial Marks |
| :---: | :---: | :---: | :---: |
| 4(c) | $\frac{20}{81} \mathrm{oe}$ |  | M2 for $\frac{2}{9} \times \frac{4}{9}[\times 2]+\frac{2}{9} \times \frac{2}{9}$ oe or M1 for $\frac{2}{9} \times \frac{4}{9}$ or $\frac{2}{9} \times \frac{2}{9}$ oe |
| 4(d) | $\left(\frac{5}{9}\right)^{n-1}[\times] \frac{4}{9}$ oe final answer | 2 | M1 for $\left(\frac{5}{9}\right)^{n-1}$ seen |
| 5(a) | 27.3 or 27.32 to 27.33 | 5 | M4 for $\tan [\angle A C D]=\frac{83.2}{\frac{83.2}{\tan 38}+54.5}$ oe <br> or <br> $\mathbf{M} 3$ for $[A C=] \frac{83.2}{\tan 38}+54.5 \mathrm{oe}$ <br> or for <br> [CD =] <br> $\sqrt{54.5^{2}+\left(\frac{83.2}{\sin 38}\right)^{2}-2(54.5)\left(\frac{83.2}{\sin 38}\right) \cos (180-38)}$ <br> oe <br> or <br> M2 for $[A B=] \frac{83.2}{\tan 38}$ oe or for $[B D=] \frac{83.2}{\sin 38}$ oe <br> or M1 for $\tan 38=\frac{83.2}{A B}$ oe or $\sin 38=\frac{83.2}{B D}$ oe |
| 5(b) | Centre marked at midpoint of $F G$. and Angle in a semi-circle is 90 | B2 | B1 for marking the centre at mid-point of FG |


| Question | Answer | Marks | Partial Marks |
| :---: | :---: | :---: | :---: |
| 5(c) | 10.8 or 10.81 to 10.82 | 7 | B2 for 72 <br> or M1 for $\frac{180}{4+5+6}[\times 6]$ <br> and, for triangle PQR <br> B4 for [angle $\mathrm{R}=$ ] 82.8 or 82.81 to 82.83 <br> or $\mathbf{B 3}$ for $[\cos R=] \frac{5}{40}$ oe or better <br> or M2 for $\frac{4^{2}+5^{2}-6^{2}}{2 \times 4 \times 5}$ <br> or M1 for $6^{2}=4^{2}+5^{2}-2 \times 4 \times 5 \times \cos R$ <br> After $\mathbf{0}$ scored for triangle $\mathrm{PQR}, \mathbf{S C 1}$ for $\begin{aligned} & {[P=] 55.8 \text { or } 55.77 \text { to } 55.78 \text { or }} \\ & {[Q=] 41.4 \text { or } 41.40 \text { to } 41.41} \end{aligned}$ |
| 6(a) | A 9 | B1 |  |
|  | $4 n-11$ oe final answer | B2 | B1 for $4 n-k$ or $j n-11$ oe $j \neq 0$ |
|  | B 55 | B1 |  |
|  | $2 n^{2}+5$ oe final answer | B2 | B1 for any quadratic or second differences $=4$ |
|  | $\text { C } \quad \frac{6}{2187} \text { oe }$ | B1 |  |
|  | $\frac{n+1}{3^{n+2}}$ oe final answer | B3 | B2 for $3^{n+2}$ oe <br> OR <br> B1 for $3^{n+k}$ seen oe <br> B1 for $n+1$ as the numerator of a fraction |
| 6(b) | 331 cao | 1 |  |
| 7(a) | 226 nfww or 226.2 to 226.3[0] nfww | 4 | M1 for mid-points soi <br> (217.5, 221.5, 229, 239, 254) <br> M1 for use of $\Sigma f m$ with $m$ in correct interval including both boundaries <br> M1 (dep on 2 nd M1) for $\Sigma f m \div$ $(9+14+14+2+3)$ |
| 7(b) | Blocks with heights 2.8, 1.4, 0.2 and with correct widths | 3 | B1 for each correct block <br> If 0 scored, $\mathbf{S C 1}$ for two correct frequency densities soi |


| Question | Answer | Marks | Partial Marks |
| :---: | :---: | :---: | :---: |
| 8(a)(i) | $\begin{aligned} & \frac{2}{3} \pi(3.6)^{3}+\frac{2}{3} \pi(5.4)^{3}+ \\ & \pi(3.6)^{2} \times 6.5 \end{aligned}$ | M3 | M1 for either $\frac{2}{3} \pi(3.6)^{3}$ or $\frac{2}{3} \pi(5.4)^{3}$ M1 for $\pi(3.6)^{2} \times 6.5$ |
|  | 692.1 to 692.2... | A1 |  |
| 8(a)(ii) | 33.6 or 33.60 to 33.62 | 4 | M3 for $\left(\frac{0.6}{3.6}\right)^{3} \times 692 \times 10.49 \mathrm{oe}$ <br> or M2 for $\left(\frac{0.6}{3.6}\right)^{3} \times 692$ oe <br> or M1 for $\left(\frac{0.6}{3.6}\right)^{3}$ or $\left(\frac{3.6}{0.6}\right)^{3}$ oe <br> If 0 scored, SC1 for their volume $\times 10.49$ |
| 8(b)(i) | $12 \pi$ final answer | 2 | M1 for $\frac{216}{360} \times 2 \pi \times 10$ oe <br> After $\mathbf{0}$ scored <br> SC1 for final answer $8 \pi$ or $12 \pi+20$ |
| 8(b)(ii) | 302 or 301.5 to 301.6... | 4 | M1 for $2 \pi r=$ their $(\mathbf{b})(\mathbf{i})$ oe or for $\frac{216}{360} \times \pi \times 10^{2}=\pi \times r \times 10$ oe and M1 for $[h=] \sqrt{10^{2}-\text { their } 6^{2}}$ oe and M1 for $[V=] \frac{1}{3} \pi(\text { their } 6)^{2} \times($ their 8$)$ |
| 9(a)(i) | -20 | 1 |  |
| 9(a)(ii) | $\frac{x+3}{2}$ oe final answer | 2 | M1 for $x=2 y-3$ or better or $y+3=2 x$ or better or $\frac{y}{2}=x-\frac{3}{2}$ or better |
| 9(a)(iii) | 125 | 2 | M1 for $\mathrm{g}(64)$ or $2\left(4^{2 x-1}\right)-3$ |
| 9(b) | 2.5 oe | 2 | M1 for $2(2 x)-3=7$ or better |
| 9(c) | $2 x^{2}+4 x-11$ final answer | 3 | B2 for $2 x^{2}$ and either $+4 x$ or -11 in final 3 term answer or for correct answer seen then spoiled or M1 for $2 x^{2}-3+2(2 x-3)-3[+1]$ |


| Question | Answer | Marks | Partial Marks |
| :---: | :---: | :---: | :---: |
| 9(d) | 1.5 oe | 2 | M1 for $4^{2 x-1}=4^{2}$ or better |
| 9(e) | $\begin{aligned} a & =3 \\ b & =4 \\ c & =-59 \\ d & =-20 \end{aligned}$ | 3 | B2 for 3 correct values or for correct unsimplified expanded expression or for simplified four-term expression of correct form with 3 terms correct or <br> B1 for 2 correct values or for correct expansion of one pair of brackets with at least 3 out of 4 terms correct. |
| 10(a)(i) | $(15,6)$ | 2 | B1 for each |
| 10(a)(ii) | $\binom{3}{24}$ | 1 |  |
| 10(a)(iii) | 13.6 or $13.60 \ldots$ | 2 | M1 for $(-11)^{2}+8^{2}$ oe |
| 10(b)(i) | $\begin{aligned} & \mathbf{a}+\frac{3}{5}(\mathbf{b}-\mathbf{a}) \\ & \text { or } \mathbf{b}+\frac{2}{5}(\mathbf{a}-\mathbf{b}) \end{aligned}$ <br> leading to $\frac{2}{5} \mathbf{a}+\frac{3}{5} \mathbf{b}$ with no errors | M3 | M2 for $[\overrightarrow{M R}=] \frac{3}{5}(\mathbf{b}-\mathbf{a})$ oe or $[\overrightarrow{N R}=] \frac{2}{5}(\mathbf{a}-\mathbf{b})$ oe or M1 for $\overrightarrow{M N}=\mathbf{b}-\mathbf{a}$ or $\overrightarrow{N M}=\mathbf{a}-\mathbf{b}$ or a correct route for $\overrightarrow{O R}$ |
| 10(b)(ii)(a) | $k=5, c=10$ | 4 | $\begin{aligned} & \text { B2 for } c=10 \\ & \text { or } \mathbf{M 1} \text { for } c\left(\frac{2}{5} \mathbf{a}+\frac{3}{5} \mathbf{b}\right)=\mathbf{b}+4 \mathbf{a}+k \mathbf{b} \text { oe } \\ & \text { or for } \frac{2}{5} c=4 \\ & \text { and } \\ & \text { M1 for } \frac{3}{5} \times \text { their } c=k+1 \end{aligned}$ |
| 10(b)(ii)(b) | $3 \mathbf{a}+6 \mathbf{b}$ final answer | 1 | FT $3 \boldsymbol{a}+($ their $k+1) \boldsymbol{b}$ |
| 11(a) | $3 x^{2}-8 x-3$ | 2 | B1 for two terms correct or correct answer seen |


| Question | Answer | Marks | Partial Marks |
| :---: | :---: | :---: | :---: |
| 11(b) | $3 x^{2}-8 x-3=0$ | M1 | FT their part (a) |
|  | Correct method to solve their 3term quadratic $\begin{aligned} & (3 x+1)(x-3)[=0] \\ & \frac{-(-8) \pm \sqrt{(-8)^{2}-4(3)(-3)}}{2(3)} \end{aligned}$ | M2 | M1 for $(3 x+a)(x+b)[=0]$ where $a b=-3$ or $3 b+a=-8$ or for $\sqrt{(-8)^{2}-4(3)(-3)}$ or for $\frac{p \pm \sqrt{q}}{r}$ where $p=-(-8)$ and $r=2(3)$ seen or for a correct method for solving a 2 -term quadratic |
|  | $\begin{aligned} & (3,-18) \\ & \left(-\frac{1}{3}, \frac{14}{27}\right) \end{aligned}$ | B2 | B1 for one correct point or for two correct $x$ values, <br> or M1 for substitution of their $x$-values into $y=x^{3}-4 x^{2}-3 x$ shown |


| Question | Answer | Marks | Partial Marks |
| :---: | :---: | :---: | :---: |
| 11(c) | $(3,-18)$ minimum with reason $\left(-\frac{1}{3}, \frac{14}{27}\right)$ maximum with reason | 3 | Reasons could be e.g. <br> 1. A reasonable sketch of a positive cubic <br> 2. Correct use of 2 nd derivative $=6 x-8=10$, $10>0$, so $(3,-18)$ is a minimum oe. <br> 2nd derivative $=6 x-8=-10,-10<0$ <br> so $\left(-\frac{1}{3}, \frac{14}{27}\right)$ is a maximum oe. <br> 3. Evaluates correctly values of $y$ on both sides of both correct stationary points <br> 4. Finds gradient on each side of both correct stationary points. <br> B2 for 1 correct with a reason for that stationary point <br> or for both $x$-values correct with correct conclusions and reasonable sketch of a positive cubic, <br> or for correct substitution of both of their $x$-values into their second derivative shown, <br> or substitution shown for one $x$-value either side of both of their stationary points to find the gradients. <br> Or <br> M1 for showing [2nd derivative $=$ ] $6 x-8$ <br> or substitution shown for one $x$-value either side of one of their stationary points to find the gradients. <br> or for reasonable sketch of positive cubic. |

